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Verbal and Mathematical Thinking: Comparative Analysis of Some Issues

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The universe is written in mathematical language (Rene Descartes)

Abstract

This paper presents how mathematical ideas can be used to go deeply into the study of verbal cognition. Researchers in human psychology do not have an answer to the question on similarity in mechanisms of mathematical and verbal reasoning in human. The arithmetic reasoning captures deeply important properties of the world which human brain represents also in verbal cognition.

Analysis of the properties of verbal thinking in comparation with mathematical cognition is an important topic in applied linguistics and cognitive psychology.

Keywords: verbal and mathematical cognition, prime numbers, order of operations, number sequence, deep structures, metaphor, proposition, word order

1. Introduction

Methodological issue.

Human language and mathematics represent discrete and continuous properties of the real world. Human brain performs mathematical and verbal operations. The perceptual – articulatory and conceptual – intentional systems of language interact with the computational system at its interfaces. (Marc Hauser W, Tecumseh Fitch. 2010)

Human cognition is the origin of the reliability of mathematics when applied to real world. It refers also to linguistics in a broader sense. Comparative analysis of mechanisms underlying the verbal and mathematical cognition has a methodological and practical significance.

The functional relationship between number and language is evident from their relationship in the brain, some basic aspects of numerical cognition depend on language. But cognitive development reflects neural organization in separating language from number. (Rochel Gelman, 2005) Multiple cognitive processes are shared between mathematical and verbal reasoning. Association and differences between numerical and verbal processing point to shared mechanisms as well as mechanisms that are distinct between verbal cognition and mathematics. The current research topic aimed to explore underlying mechanism of the overlap between mathematical and verbal cognition.

Mathematics is not an arbitrary game with arbitrary rules, rather it must agree with how our thinking, and then our speaking and writing, proceeds. (Marcus Giaquinto, 1983)

Basic elements of human verbal and mathematical reasoning, order of these prime elements, mental operations employed in verbal and mathematical structures are the object of analysis in this paper. Main methodological approach of this research is a structural analysis in combination with functional description of operations with above named basic elements and complex structures. According to William Brownell, arithmetical meanings include understanding of fundamental operations. (Kilpatrick & Weaver, 1977) This idea seems to be closely related to theory of mental schemas, concepts and its transformations underlying human perception and language.

Relationship between a signifier and signified, relationship between an object and its representation in a sign (symbol) constructed by the child, is reflected in both human numerical and verbal cognition. But there is

significant difference in construction of mathematical knowledge and symbolic processes by which child represents actions. (Smock.D, 1981) Kurt Gödel believed that objective mathematical reality could be perceived in a manner analogous to sense perception. (Wang, 2001) Mathematical and verbal structures embedded in human mind, operations with these structures present methodological significance for analysis of human general cognitive mechanism.

Arithmetical meanings are associated with semantic meanings, arithmetic operations involve an association with language syntax. There is important to determine basic elements of verbal deep structures, otherwise to find elements like prime number which has no factors besides itself, and divisible by itself and 1. Idea of prime numbers closely connected with deep structures in Universal grammar. (Chomsky, 1965) Primes serve as a building block of deep structures. Composite numbers made of primes have its analogy in verbal cognition as a complex semantic structure, proposition.

Theory of prime numbers must provide non-traditional description of evidence of deep structures in Universal grammar (Terence Tao, 2009). There is basic argument to employ mathematical concepts of prime number and order of operations in linguistics. This methodological assumption is derived from specific topic of research.

Ideas of prime factorization and order of operations are allowable within the paradigm of research. Set of sequence introduced in number theory is useful to think of a sequence as a function from the set of words in human mental vocabulary. The order of operations has methodological significance to describe basic rules and constraints for combining structural and semantic features into words and combining words into a potentially infinite number of phrases and sentences.

2. Prime Factorization in Verbal and Mathematical Thinking

Factorization, besides its mathematical value, has important methodological significance in linguistics. Semantic structures, propositions as a deep structures have common factors differing in structural and functional roles. This is main reason to apply an idea of prime factorization to structure analysis (word, sentence, discourse) in linguistics by reducing these structures to basic building blocks. Deep semantic structures present an object of analysis in terms of greatest common factor (GCF), greatest common divisor (GCD) and least common multiple (LCM). GCF as a largest of common factors in numbers will be effective to determine similar sememe in word semantics or similar proposition in complex semantic structures. GCF is applied to description of common semantic component determining semantic volume of words in different languages.

Table 1

		Оюутан	Student	Студент
1	to study	+	+	+
2	university/college	+	+ -	+
3	gender	-	-	+

Parts of speech in linguistics are the object of analysis by using prime factorization. Common semantic (also syntactic) properties, main (great) common semantic factor serve as a basis of associative network of words in human mental vocabulary. Classification of verb and noun classes as an expressions of static and dynamic properties, classification of nouns present product of mental operations similar to factorization. Factorization in human mind must be based on denotative, objective and also connotative, subjective features. There is some difference between factorization in verbal cognition and mathematics. Reference of the word “dog” is associated with the class of home animals and also has connotative link with “human friend”.

GCF is used to simplify common factors reducing largest common factor.

GCF of 12: 1, 2, 3, 4, 6, 12
of 16: 1, 2, 4, 8, 16 ⁄ can be simplified: $4\frac{12}{16}$ ($\div 4$: $\frac{3}{4}$; $\div 4$) (1)

In the same way, a simplification idea will be used to reducing complex semantic structures to basic primes. GCF in combination with Venn diagram must be applied to classification of words in human mental vocabulary. If the greatest common factor will be the product of the numbers in the intersection, words with same semantic and syntactic value intersect.

According to Venn diagram, GCF (24, 30) =2x3=6, prime factorizations: $24 = 2^3 \times 3;\ 30 = 2 \times 3 \times 5$

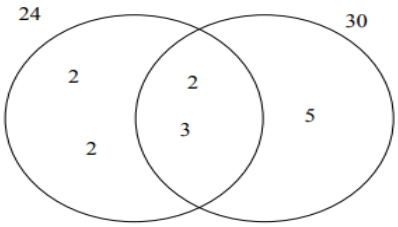


Figure 1

In that's way idea of GCF is good reason to describe phenomena of intersection in classification of parts of speech, classes of words in human mental vocabulary. In Mongolian language common multifunctional words, for example, attributive words serving as adjective and adverb in different positions intersect in one set.

хурдан	морь	racing horse
(fast, quickly)	гүйх	to run faster

Great common divisor (GCD) also presents effective tools to be used with Venn diagram.

$$(gcd(42,56)=14,\ \text{therefore},\ \frac{42}{56}=\frac{3\cdot 14}{4\cdot 14}=\frac{3}{4}) \tag{2}$$

It means that factorization and Euclidean algorithm reducing the size of the larger integer until the two identical integers are effective to reduce the size of larger structures to identical basic mental structures.

GCF and LCM in connection with Venn diagram have an interpretation in terms of intersection $A \cap B$ in Boolean algebra. LCM is inductive way for determination of common elements, components in human semantic vocabulary, complex syntax structures. LCM more applicable to analysis of complex structure because of LCM must be a composite number when GCF is only prime number. LCM also has direct relation to Venn diagram

$$\text{due to LCM } (a,b) = \frac{a\times b}{\gcd(a,b)} \tag{3}$$

LCM used to describe an effect of semantic transformations creating similar semantic value in a word, sentence making an equivalence of these structures. In that's way LCM is effective for an analysis of cohesion at word and sentence levels in different languages.

multiples of 3:3, 6, 9, 12, 15, 18 …
multiples of 5:5, 10, 15, 20, 25 … LCM = 15

If multiplication is applicable for example, to metaphor, LCM is a way to find basic common semantic component or building block in metaphors in one and different languages. Examples:

Brian went from the car to the store.

The country went from conflict to peace building.

Brain went from sad to happy.

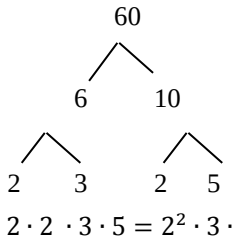
Дорж малчнаас эзэн суув. (Dorj went from the cowboy to the owner).

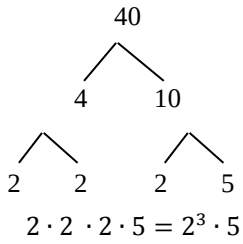
went	from the car	<	went from the car.
	to the store		went to the store.
went	from conflict	<	went from conflict
	to peace building		went to peace building
went	from sad	<	went from sad
	to happy		went to happy
went	from the cowboy	>	went from the cowboy
	to the owner		went to the owner

In third and fourth sentences, cohesion between the verb and the noun (with preposition) presents a case of multiplication leading to semantic transformation. LCM is to describe parallel semantic structures containing similar concluding propositions (with same global meaning) differing in sequence of micro propositions as building components of main proposition. The GCF as a product containing the smallest integer of factors is effective tools to be used in component analysis of syntax structures on the principle of binarity.

GCF divides up a sentence into immediate constituents and this process continues until irreducible constituents are reached.

Prime factorization of 40 and 60.





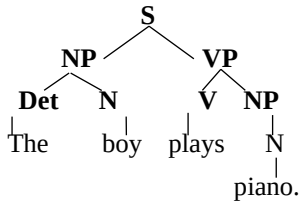
List of ALL of the prime factors:2, 3 & 5

Complete list of factors:

$LCM = 2^3 \cdot 3 \cdot 5 = 120$

Figure 2. (a) (b)

Following sentence trees present a case to apply GCF.



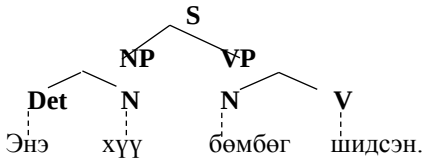


Figure 3. (a) (b)

Sequence of factors in above named sentences:

Det · N · V · N

Det · N · N · V

Following trees present another case to use GCF in the syntax:

Figure 4. (a) (b)

Above named sentence trees, differing in a sequence of factors serve as argument to apply GCF in linguistics in combination with the immediate constituent and the grammatical tie. GCF also must be employed in combination with LCM.

LCM must be used to determine basic immediate constituents of syntax structures as adjective-noun, verb-adverb, verb-object, excluding initial binary subject-predicate unit.

It means that If GCF is more useful for description of sentence building from prime components, LCM could be effective for determination of largest common component serving as a basic building block of syntax structure. In that's way GCF is applied to analysis of complex syntax structures.

Complex sentence trees:

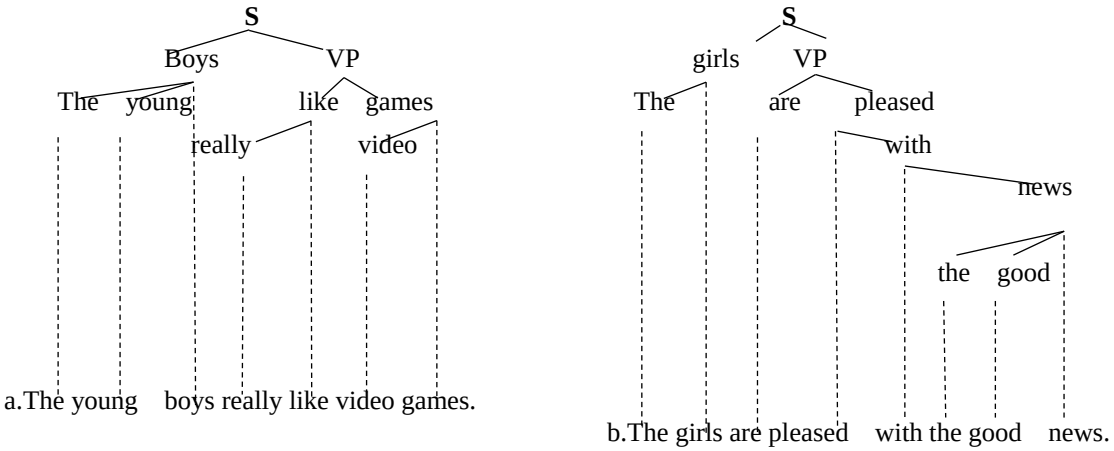


Figure 5. (a) (b)

Applying factorization in linguistics means a new opportunity to extend the immediate constituent (IC) analysis and Universal grammar theory in new dimensions.

3. Sequence of Constituents and Order of Operations in Verbal and Mathematical Thinking

Order of operations and number sequence reflect incredible example of symmetry in nature. Arithmetic and geometric sequences, Fibonacci sequence contain an idea what can be used in variety of phenomena in nature and life.

Fibonacci sequences (1.1.2.3.5.8.13.21.34.55.89.144…) appear in biological setting are arrangement of leaves on a stem, in genetics (human X chromosome inheritance tree), golden ratio in animals (between the forearm and the hand). Arithmetic and geometric progressions can be used in description of logical operations underlying simple and complex verbal structures. Some specific type of discourse (complex semantic structure) as a poetry, music and paintings, symmetry in music and lyrics where something is coming down and then going up in precisely the same way present phenomena to be described in terms of Fibonacci sequences.

Number sequence, particularly the sequence of prime numbers must have an analogy in a sequence of semantic, deep structures in verbal cognition. Sequence of verbal structures must be modeled in horizontal or vertical dimensions.

adj 1 adj 2 adj 3 adj 4 N

Хуучин жижиг хүрэн модон байшин

(Old small brown wooden house)

adj 1 adj 2 adj 3 adj 4 N

New Japanese red sport car

Attributive structures with same meaning may differ in sequence of components.

Хотын хүү — The city boy .

Boy from the city.

New York chrysler building – Chrysler building in New York.

These parallels with sequence difference is not similar to "2+3=3+2", (4) but there is a point for comparison.

We consider that a sequence of expressions with intrinsic and extrinsic properties of an object have its origin in human perceptual system. Expression of attributive relation is a case for analysis of sequence of semantic primes in analogy with a sequence of prime numbers:

Size color material

Намхан шар модон байшин.

Size shape color material

Жижиг дөрвөлжин цэнхэр модон хайрцаг.

Differences in human perception of an object and its properties are reflected in a sequence of deep and surface structures in verbal cognition. Some languages underspecified color, some-shape, and some-motion. Gradable adjectives such as "big" tend to highlight dimensions of relations between objects as well as intrinsic dimensions. (Catriona Silvey, 2015)

Intrinsic dimensions are reflected in a sequence of attributive components in relation to noun (object or subject). Attributive expressions with plural references must be interpreted as a sequence of words similar to complex number addition

$$\big((a+bi)+(c+di)=(a+c)+(b+d)i\big) \tag{5}$$

a b m c d n

Өндөр төмөр хашааны намхан цэнхэр хаалга

$$(a+b)m+(c+d)n \tag{6}$$

c d n a b m

Намхан цэнхэр хаалгатай өндөр төмөр хашаа

$$(c+d)n+(a+b)m \tag{7}$$

a b c d e f j

Old large wooden house with extra bedroom decorating.

$$(a+b+c)d+(e+f)\cdot j \tag{8}$$

In linguistics phrase, clause and sentence are the sum of internally connected components in horizontal dimension. Simple sentence must be interpreted as prime or composite in horizontal dimension.

Discourse as a sum of propositions based on logical hierarchy is an illustration of sequence rules similar to number sequence in vertical position. Discourse in vertical dimension is interpreted as a composite.

Composite

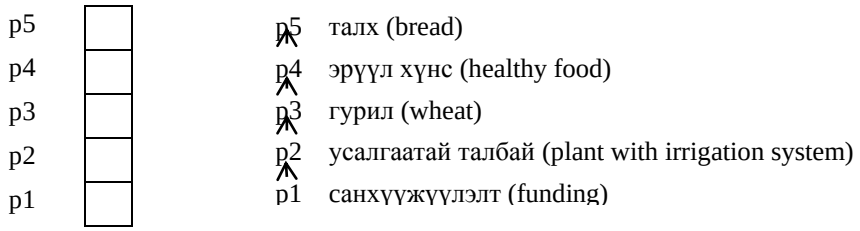


Figure 6

Prime factorization is effective for analysis not only in case of symmetry, but also in case of broken symmetry in verbal structures and non-verbal cognition.

Geometric sequence and progression is more applicable to analysis of fractal and other semantic structures breaking symmetry. An imaginary component in complex number addition is applicable for description of common effect of denotative and connotative components in attributive structures with double references. Addition or subtraction from left to right, multiplication or division from left to right present analogy with operations taken place in verbal cognition.

Phrases: Хар цамц (black shirt), Хар шөл (meat soup), Хар шөнө (dark night), Хар санаа (hostility) are example of addition and multiplication in verbal structures.

Phrases - хар шөнө (dark night, no moon), сэтгэл онгойх (to open up inner thoughts) are characterized by strength of cohesion between components multiplied by creating new complex semantic value.

Linear and non-linear thinking has different expressions in verbal cognition. Non-linear thinking leads to creating verbal metaphors. According to Roger Antonsen, equation $x + x = 2x$ has two different perspectives. (Roger Antonsen, 2016). It's analogy we can find in metaphorical expression (Од харвах - Звезда стреляет – Star shooting as a path of a meteoroid) in Mongolian and Russian having different interpretations as a product of non-linear thinking.

Operations at phrase, sentence, complex sentence levels have a similarity with prime and complex number operations. Subject and verb (predicate) group in sentence, clauses in complex sentence with subordinate relations must be interpreted in similar way as a complex number operation. Word order in typologically different structures (S-O-V, S-V-O) present specific phenomenon for analysis in terms of PEMDAS:

Ах ном авсан [S+(O+V)]

The boy caught the ball. [S+(V+O)]

PEMDAS as a complex idea including parentheses, exponents, multiplication, division, addition and subtraction must be applied to analysis of an order of operations in a sentence, a discourse.

Order of addition and multiplication, subtraction and division, parentheses ( ), brackets [ ], braces { } exponents presents an analogy in verbal operations.

{The woman (whose dress rustled) [when she walked] sat down beside me}

{The woman sat down beside me}.

In description of verbal structures at different levels is significant to consider an order of components and operations despite at similar result or sum.

$$1+1+4=2\times3$$

$$1+4+1=3\times2 \quad (9)$$

Complex structures with differences between subject-extracted and object-extracted structures present an phenomenon to be described by using PEMDAS.

a. Subject-extracted relative structure

The reporter [who ____attacked the senator] admitted the error.

b. Object-extracted relative structure

The reporter [who the senator attacked_____] admitted the error. (Gregory Scontras, 2015)

Number operations present an instrument to explain a misunderstanding of verbal structures, discourse. Misunderstanding like (3+8×5=55("43" is correct response) is taken place in interpretation of sentence, text in neurolinguistics research. So there is important to conduct parallel analysis of misinterpretation in verbal communication and misunderstanding of number operations.

PEMDAS presents an effective instrument of analysis at discourse level. Following propositions must be described in terms of PEMDAS.

1. Гэр хорооллын оршин суугчдыг орон сууцтай болгох – public housing
2. Байрны зээлийг бага хүүтэй олгох – decreasing loan percentage

3. Эрчим хүчний хангамжийг шийдэх – improving electricity delivery
4. Утаа арилж агаарын бохирдол буурах – cleaning up the smoke and reducing air pollution
5. Хүн амын эрүүл мэндэд учрах эрсдэл буурах – decreasing health risks for people

Proposed sequence of analysis by using PEMDAS

Propositions {1; 5}
Propositions (2; 3)
Propositions [(2+3)· 1]
Propositions [4+5]
Concluding proposition: $5\times\{[1+ (2+3)]\times4\}$ (10)

PEMDAS analysis leads to determine a super proposition as concluding proposition in discourse, to describe an equivalence at macro semantic discourse level.

Coherence of propositions in a discourse in some cases originate a roughness. This is an example of common effect of denotative and connotative meanings, of possible contradiction between objective and subjective meanings.

Metaphorical thinking, humor serve as a way to roughness in verbal thinking. So as an effect of roughness, a fractal in verbal expressions in some way must create non-linear verbal structures in 3D and 4D. PEMDAS is suitable for analysis of surface roughness of discourse at the deep level.

4. Discussion and Conclusion

Comparative analysis of some issues in human mathematical and verbal reasoning leads to deep understanding of human neurocognitive mechanism.

Despite the fact that numerical concepts have an ontogenetic origin independent of language, number sequence and order of operations in mathematics must have close connection with the order of operations in verbal thinking.

Prime factorization presents an alternative way to determine universal basic components or units of verbal structures. GCF, GCD and LCM serve as an effective instrument to describe sentence building and discourse interpretation.

Order of number operations are associated with order of components in verbal syntax. Semantic structure of sentence and discourse, cohesion and coherence between units of sentence or discourse are based on an order of operations similar with the order of operations in mathematics.

Arithmetic operations as an addition and multiplication have its analogy in verbal structures. Factorization and order of operations support more deep analysis of universal associative mechanism underlying mathematical and verbal operations.

Ideas of prime factorization, number sequence and order of operations involving basic mental schemas and concepts must be employed in analysis of deep structures and mental transformations. In that's way some finding of this study are tied to psychology and neurocognitive science and serve to extend some notions of Universal grammar.

Our research comprises the application of effective mathematical instruments with main emphasis on deductive reasoning and has a positive impact on development of the techniques associated with quantitative research.

Employed by us mathematical instruments must be used to enable more creative approach leading to universal, unifying results in analysis of structural features that are common to natural languages.

Comparative analysis supports to an idea that linguistic and mathematical creativity serve as a basis of generalizability in human cognitive operations and help to develop an in-depth knowledge about the topic under consideration.

This research paper has limitations that necessarily raise questions for future research to determine the full extent of the relationship between verbal and mathematical cognition.

Above presented preliminary suggestions represent a challenge to the traditional ways of linguistic analysis contributing to the development of interpretive paradigm of linguistics research.

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