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A Simple and Effective Algorithm to Calculate the Hurst Exponent

Luis Fernando Llano-Ferro[1]

[1] Private Consultant, Bogotá, Colombia

Correspondence: Luis Fernando Llano-Ferro, Private Consultant, Bogotá, Colombia. E-mail: llanoluisfernando@gmail.com

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Abstract

In 1951 the British hydrologist Harold Edwin Hurst published the paper: "Long Term Storage Capacity of Reservoirs". The paper introduced the Hurst Exponent. It was a tool to analyze river flows to help determine the appropriate size of a reservoir. In 1968 Benoit B. Mandelbrot and John Van Ness published the seminal paper "Fractional Brownian Motions, Fractional Noises and Applications" In effect this paper broadened the applications of the Hurst Exponent to time series in physics, geophysical sciences, medical science, biology, information sciences, economics, finance, etc. The present paper presents an algorithm to calculate the Hurst Exponent. It is based on the "bootstrap method" introduced by Bradley Efron in 1979. It is simple to implement, and permits the assignment of measures of accuracy to the estimand.

Keywords: time series, Hurst exponent, Monte Carlo simulation, bootstrapping

1. Introduction

In 1951 the British hydrologist Harold Edwin Hurst published the paper "Long Term Storage Capacity of Reservoirs" in the Transactions of the American Society of Civil Engineers. Hurst worked for 62 years for the Egyptian government studying the Nile. Its procedure was intended as a tool to analyze river flows to help determine the appropriate size of a reservoir. In 1968 Benoit B. Mandelbrot and John Van Ness published the paper "Fractional Brownian Motions, Fractional Noises and Applications". In effect, this seminal paper, published in the Review of the Society for Industrial and Applied Mathematics (SIAM), broadened the applications of the Hurst Exponent to time series in physics, geophysical sciences, medical science, biology, information sciences, economics, finance, etc. The present paper presents an algorithm to calculate the Hurst Exponent. It is based on the "bootstrap method" introduced by Bradley Efron in 1979. It is simple to implement, and permits the assignment of measures of accuracy to the estimand.

2. Hurst's Algorithm to Calculate the Hurst Exponent

For explanatory purposes, of the original Hurst procedure to analyze river flow time-series, let us take a time series with 64 data points. This series is divided into a total of 63 separate blocks. One block is 64 data points long, 2 are 32 points long, 4 are 16 data points long, etc.

a. The first step of the procedure is to take the average of the 64 point series, and center the data; subtracting the average. Then the data is assigned to the 63 blocks.

b. For each block we calculate the Standard Deviation, and the cumulative sum of the data. For example, for a 4 data point block if the centered numbers in the block are: 3,-6,4,and 9,the cumulative sums are: 3,-3,4,13. The maximum of the cumulative sums is 13 and the minimum is -3. The Range is defined as Maximum minus Minimum of the cumulative sum. (In this case Range=13-(-3)=16). The Rescaled Range was defined by Hurst as the Range divided by the Standard Deviation. For this example the Standard deviation is 6.245. Therefore, the Rescaled Range is 16/6.245 = 2.562. (by-the-by, Hurst calculated the Standard Deviation dividing by n, instead of by (n-1)).

c. Take the average of the Rescaled Range of the same size blocks. The size 2 blocks has 32 members, the size 4 blocks 16, etc. Let us call these 6 averages: $Y_{2,,}$ $Y_{4,,}$ $Y_{8,,}$ $Y_{16,,}$ $Y_{32,,}$ Y_{64}

d. Take the logarithm of the Y,s and the X,s. Hurst used base 2 logarithms, simply because the X axis readings would then become exactly, for our example 1,2,3,4,5,and 6. In the early 1950´s digital computers were not prevalent, and – presumably - Hurst expected that these calculations would be carried out by hand. Taking base-2 logarithms would facilitate calculations.

e. Calculate the Ordinary Least Squares Regression of Log(Y) on Log(X). Nowadays natural logarithms are used. The Hurst Exponent is the slope of the regression.

A draw back of the Hurst procedure is that the length of the data has to be adjusted to powers of 2. If we had say 120 points, in our example, we would have to discard 56 data points, or execute several runs, distributing the data in different forms.

3. Proposed Algorithm to Calculate the Hurst Exponent

The algorithm proposed is as follows:

f. Execute step a. The series can be of any length.

g. Randomly select 2 integers *a*, and *b* in the range 1 to n, where n is the number of data points. If *a* is greater than *b* the numbers are interchanged. If the absolute difference of the random integers is 1 or less, discard them, and generate new random integers. What we are doing here is "bootstrapping": random sampling with replacement.

h. For the block of size *a*-*b*, and in the data range *a* to *b*, calculate the standard deviation, the range and the Rescaled Range.

i. Create the arrays $Y_i = Log_e(Rescaled\ Range_{i.})$, $X_i= Log_e(b-a)$.

j. Repeat steps g,h and i as desired. From 1000 to 5000 repetitions are recommended..

k. Calculate the regression of X on Y. The slope of the Regression is the Hurst Exponent.

4. Conclusion

This paper has presented an algorithm to calculate the Hurst Exponent. It is based on bootstrap methods. It is simple and effective. It can be applied to time series of any length.

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