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A Completely Nonlinear Approach of Multilevel Regression for Detecting Turning Points: Developing an Alternative Platform With an Application of TIMSS Data

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Abstract

There have been various traditional methods to identify turning points to establish nonlinear relationships. These methods use a linear approach (i.e., traditional piecewise regression) to seek a nonlinear relationship. The present study aimed to introduce a completely nonlinear approach as an alternative platform to identify turning points. This alternative approach was also multilevel to work with data hierarchy for the identification of turning points. The United States sample (8776 students from 287 schools) in the 2019 TIMSS (Trends in International Mathematics and Science Study) was applied to this alternative approach to identify turning points in the relationship between mathematics achievement and mathematics enjoyment with control of student and school characteristics. This alternative approach performed well, successfully revealing positive but differential effects on mathematics achievement across different degrees of mathematics enjoyment.

Keywords: completely nonlinear approach, turning points, multilevel, TIMSS

1. Background

Statistically, there is a turning point concerning the effects of X (independent variable) on Y (dependent variable) when a value of X is identified before and after which the effects of X on Y are statistically significantly different. The Merriam-Webster dictionary echoes this notion, calling a turning point as a point at which a significant change occurs. Monitoring changes, especially significant ones, is a major task in many fields, demanding the detection of turning points. Psychologists, for example, attempt to identify behaviors that emerge at a certain age. Child development is all about a significant change in behaviors when children reach a specific age (e.g., Cole, et al., 2005). For another example, educators who believe in a relationship between study time and academic performance search for a threshold study time at which the effects peak on academic performance (e.g., Ma, et al., 2013). With such a background, we conducted the present study to advance statistical techniques for identifying a turning point. We intended to develop an alternative analytical platform to the traditional approaches with several clear methodological advantages.

2. Traditional Piecewise Regression Approach

There have been various methods in practice to detect turning points. The present study summarized the commonly used ones, each with an example of application from the research literature. Visual inspection (the eyeball method) is often applied to identify the number of segments as well as the position of turning points (Crawley, 2012). In some cases, naked eyes work well to pinpoint turning points between segments. Generally, a scatter plot offers a good picture of the relationship, revealing potential turning points. When the pattern of change underlying the data is subtle, it is difficult to apply the eyeball method. Hernández-Lloreda, et al. (2004) studied the change in the relationship between mothers and infants during different periods of infancy. The eyeball method suggested the polynomial degree appropriate to capture the nonlinear relationship in the data, helping formulate research hypotheses of the effects.

In some cases, a turning point has been naturally established that would divide data into regions (the establishment method). Life science often contains such cases. Burke, et al. (2008) offered an example of conjugal loss as an established turning point for life. Conjugal loss triggers traumatic symptoms to cause a discontinuity of wellbeing.

Many time-varying variables have nonlinear relationships with conjugal loss. For instance, depression is mild before the loss, escalates at the time of the loss, and gradually decreases after the loss.

Theories can often provide informative clues on turning points (the theory-driven method). Social science examines the relationship between humans and society, with one of the main goals being to identify how the relationship changes over time (Elliott, 2014). There are theories that offer important hypotheses about a certain relationship. For example, during the development of autonomy in emerging adults, contact with families declines. This decline is captured in the theory of separation individuation (Mahler, et al., 1975), with two crucial turning points (Sneed, et al., 2006) which can be gender specific. Age 17 represents the high rate of family contact with similar levels between men and women, whereas age 27 represents the low rate of family contact with different levels between the gender groups. Some empirical studies adopt those theory-driven turning points (e.g., Rindskopf & Sneed, 2008).

When turning points are neither observable by eyes nor identifiable by establishments or theories, exploratory work is often pursued (the data-driven method). This method relies heavily on statistical manipulations. For example, Li, et al. (2019) indicated that there is a threshold of teaching quality before and after which teaching has differential effects on children's learning outcomes such as early mathematics, language, and social cognition. Without any credible guidance for model specification, they examined multiple values of teaching quality as potential turning points, testing slopes before and after each value for statistical significance.

Statistically, the four methods discussed above apply the same procedure of piecewise regression to detect turning points. When a turning point is identified, piecewise regression is statistically very similar to regression discontinuity. The logic for the application of piecewise regression for the detection of turning points is obvious. Because it is impossible for one linear regression line to capture turning points in the case of a nonlinear relationship, linear regression is performed region by region across the values of the independent variable (thus piecewise regression). Piecewise regression is the traditional statistical approach, evolved from multiple regression, to detect turning points.

A simple multiple regression model can be used to demonstrate the framework:

$$Y = \pi_0 + \pi_1 X + \varepsilon$$

Where Y is the dependent variable, X is the independent variable that may contain a potential turning point, and ε is the error term. Meanwhile, π_0 is the intercept and π_1 is the coefficient (slope) of X, indicating the effects of X on Y. Suppose a visual inspection indicates a turning point for the effects of X on Y, then X is divided into X_1 (region below the point) and X_2 (region above the point). With data split into two segments, the equation can be expressed as:

$$Y = \pi_0 + \pi_1 X_1 + \pi_2 X_2 + \varepsilon$$

where π_1 and π_2 are different slopes of X, representing different effects before and after the turning point.

Although the traditional approach of piecewise regression is simple and popular, it is essentially a linear approach to establish a nonlinear relationship. Of course, such an analytical strategy is not inherently inappropriate. Because the traditional piecewise regression becomes difficult to handle when there are multiple turning points within X (Muggeo, 2008), there is the parsimonious practice of imposing just one or two turning points (to make piecewise regression both efficient and effective statistically). As a result, the researchers often get an oversimplified nonlinear relationship.

The traditional piecewise regression may also identify unrealistic turning points. For example, in a Likert type of measurement (e.g., 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree), a decimal has no measurement meaning, and identifying a decimal as a turning point creates confusion for social policies and practices. Finally, as the researchers pursue the identification of turning points in complex data such as data with hierarchical structure (e.g., students nested within schools), there is the need to combine multilevel modeling with techniques that identify turning points (e.g., Li, et al., 2019; Muggeo, 2008).

3. A Completely Nonlinear Approach of Multilevel Regression

The present study attempted to develop an alternative analytical platform for the detection of turning points, with two methodological purposes. First, it sought a completely nonlinear approach to establish more "naturally" a nonlinear relationship. Second, it brought a multilevel perspective (i.e., hierarchical linear modeling or HLM) into the analytical platform to detect turning points in complex data (e.g., students nested within schools). The alternative approach can take the form of a two-level HLM model. The first level focuses on individuals (e.g., students), and the second level focuses on institutions (e.g., schools).

Take the example of a data hierarchy with students nested within schools. A student-level variable of X is examined for potential turning points in terms of the effects of X (X_{nij} precisely) on the dependent variable of Y (Y_{ij} precisely). The subscription of *n* in X_{nij} indicates a Likert type of measurement scale, for example, a five-point Likert type of measurement scale (i.e., *n* = 1, 2, 3, 4, 5 with 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree). What makes the alternative approach completely nonlinear is the use of dummy coding to represent each measurement point. Specifically, the student-level model is a set of separate regressions, one for each school. It can be expressed as:

$$Y_{ij} = \pi_{1j}x_{1ij} + \pi_{2j}x_{2ij} + \pi_{3j}x_{3ij} + \pi_{4j}x_{4ij} + \pi_{5j}x_{5ij} + \sum_{p=1}^{m} \pi_{(5+p)j}x_{(5+p)ij} + e_{ij}$$

$$x_{nij} = \begin{cases} 1, for\ x = n\ (n = 1, 2, 3, 4, 5) \\ 0, for\ x \neq n\ (n = 1, 2, 3, 4, 5) \end{cases}$$

where Y_{ij} is the value of the dependent variable for student *i* in school *j*, x_{nij} are the dummy variables to indicate response points, $x_{(5+p)ij}$ (p = 1, 2, … m) are all the student-level variables to be controlled for at the student level, and e_{ij} is the error term for student *i* in school *j*.

Obviously, in the above equation, π_{1j} to π_{5j} are measures of effects at each measurement point of X. The school-level model then adjusts π_{1j} to π_{5j} for school-level variables:

$$\pi_{1j} = r_{10} + \sum_{q=1}^{n} r_{1q}z_{qj} + U_{1j}$$

$$\pi_{2j} = r_{20} + \sum_{q=1}^{n} r_{2q}z_{qj} + U_{2j}$$

$$\pi_{3j} = r_{30} + \sum_{q=1}^{n} r_{3q}z_{qj} + U_{3j}$$

$$\pi_{4j} = r_{40} + \sum_{q=1}^{n} r_{4q}z_{qj} + U_{4j}$$

$$\pi_{5j} = r_{50} + \sum_{q=1}^{n} r_{5q}z_{qj} + U_{5j}$$

where the *r* parameters (r_{10}, r_{20}, r_{30}, r_{40}, r_{50}) are adjusted effects of X for school-level variables, z_{qj} (q = 1, 2, … n) are all the school-level variables to be controlled for at the school level, and U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j} are school-level error terms.

Based on the alternative approach, the statistical significance of the coefficients r_{10}, r_{20}, r_{30}, r_{40}, r_{50} will suggest whether these measurement points, with adjustment over student-level variables and school-level variables, are turning points of X on Y. From the above HLM model, there can be three advantages to the alternative approach. First, the identification of turning points is determined with adjustment over both student-level variables and school-level variables. Second, the alternative approach for determining turning points is entirely nonlinear so that there can be "naturally" multiple turning points as long as statistically significant changes occur on those points. Finally, turning points are easy to interpret and understand concerning implications for social policies and practices because there are no decimal turning points (i.e., each turning point is precisely a measurement point). The traditional approach (of piecewise regression) may not possess these advantages. Overall, the alternative approach is simple, systematic, and naturally and completely nonlinear as far as model specification is concerned.

The above HLM model uses the Likert type of measurement scale for the independent variable, indicating a major weakness for the alternative approach. There is a possible need to collapse data when the independent variable is continuous, which loses some information. This weakness can be improved by setting up an adequate number of potential measurement points. For example, if the independent variable is measured on a continuous scale of zero to 100 (score points), ten measurement points may be considered (i.e., 0 = 0 – 9, 1 = 10 – 19, 2 = 20 – 29, and so on). If collapsing data can be considered a useful way of simplifying data as some studies do to offer a concise implication for social policies and practices, then this weakness may not even be much of a weakness.

Finally, because the alternative approach takes the form of a two-level HLM model, there are no added assumptions to those common to any HLM model. Simply put, according to Raudenbush and Bryk (2002), the assumptions are that e_{ij} is NID (normally, independently distributed) with a mean of zero and variance across schools; student-level variables are independent of e_{ij}; residual school variances ($U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}$) are NID; school-level variables are independent of $U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}$; and errors at the student and school levels are independent of each other.

4. Extension

There are possible analytical extensions of the alternative approach outlined above (i.e., the completely nonlinear approach of multilevel regression). The most straightforward is the manipulation of the random effects in the HLM model above. Obviously, the above equations of the HLM model indicate that at each measurement point the effects of X on Y are assumed to be random at the school level (i.e., with the presence of $U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}$ at the school level). This treatment obviously assumes that the effects of X on Y at each measurement point differ from school to school (i.e., at the school level). To some researchers, this treatment is quite realistic and may capture the nature of school effects. Because schools are different on many fronts, it is reasonable to have different schools showing different effects. To other researchers, the effects of X on Y may be considered similar, particularly when schools share some common characteristics (e.g., geographic characteristics because of being in the same region).

The alternative approach is flexible in that the error terms at the school level can be either present or absent based on certain assumptions on the part of educators and researchers. Statistically, estimating the fixed effects of X on Y is easier and the results are more accurate, whereas estimating the random effects of X on Y is more complex and the results are more prone to errors. While this decision is a practical and substantive one, the alternative approach can easily accommodate both demands. Of course, it is always possible to take a data-driven strategy to test whether U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j} are statistically significant or not for the purpose of decision making. Nonetheless, it is worth emphasizing that the possibility of treating the effects of X on Y to be random at any given measurement point represents another advantage for the alternative approach over the traditional approach (of piecewise regression).

5. Application

5.1 Data

Data used for this application was the United States (US) sample in 2019 TIMSS (Trends in International Mathematics and Science Study). TIMSS is an international comparison project by the International Association for the Evaluation of Education. Since 1995, TIMSS has collected and analyzed data every four years, and 2019 TIMSS was the latest assessment circle. Sixty-four countries (educational systems) participated in 2019 TIMSS. TIMSS collects data from students, teachers, and (school) principals to measure home, classroom, and school contexts, with a two-stage (schools and students) random sampling design. For this application, the US national representative sample of fourth graders was used (8776 students from 287 schools). The average age in the US sample was 10.2 years.

5.2 Variables

For this application, the outcome variable was mathematics achievement of the fourth graders. In TIMSS, the content domains include number, measurement and geometry, and data; and the cognitive domains include knowing, applying, and reasoning. For fourth graders, the content emphasis is number (including introductory algebra or pre-algebra topics), and the cognitive emphasis is knowing (see Mullis & Martin, 2017). To minimize testing time, 2019 TIMSS applied matrix sampling to generate plausible values for each student (five plausible values of mathematics achievement for each student). Since plausible values cannot be directly used as (traditional) test scores, integration of the five plausible values was done to produce a score in mathematics achievement for each student. 2019 TIMSS put each plausible value on a measurement scale with a mean of 500 and a standard deviation of 100.

Independent variables were obtained at both student level and school level. Student-level variables included student background characteristics as control variables, including gender, age, socioeconomic status (SES) (measured by the number of books at home), home language (i.e., whether speak English at home), and immigration status (i.e., whether born in the US). The variable of interest, mathematics enjoyment (referred in TIMSS to as students like learning mathematics), was also at the student level. Mathematics enjoyment was measured with nine items on a four-point Likert type of measurement scale (see Appendix A). Finally, mathematics enjoyment was a composite variable constructed from the scale of the nine items by calculating the valid average across all items (i.e., calculate the average based on those items of the scale that had valid responses).

School-level variables were also used as control variables, including school contextual variables and school climate variables. Contextual variables included school location (urban, suburban, and rural), school socioeconomic composition (measured by shortage of resources in technology, laboratory, and library), and school racial-ethnic composition (measured by school proportion of majority students). Climate variables included school resources (for mathematics education), academic pressure (i.e., school emphasis on academic success), and school disciplinary climate (i.e., school safety). School climate variables were composite variables, and each was constructed from a scale of items by calculating the valid average across all items (see Appendix A). These student-level and school-level variables are important adjustments when estimating academic achievement (see Ma, et al., 2008).

5.3 Procedures

As in the routine practice of multilevel modeling, the null HLM model was run first that included only the outcome variable (i.e., mathematics achievement). This analysis of the null model and all subsequent analyses were executed on the HLM8.0 platform (Raudenbush & Congdon, 2021). The HLM8.0 program has a function to integrate plausible values, which was applied in all statistical analyses in this study. The main result from a null model is usually the intra-class correlation (ICC) measuring the proportion of variance attributable to the second level (i.e., school level in this case). ICC = .27, indicating that a statistically significant amount of variance in mathematics achievement did come from schools. The null model also provided information for the upcoming calculation of R square (i.e., proportion of variance explained at student and school levels).

After the null HLM model, the (independent) variable of interest, mathematics enjoyment represented by a number of measurement points, was introduced. Because this variable was continuous, it was divided into five equal regions based on percentiles (i.e., 20th, 40th, 60th, and 80th percentiles) (see Appendix B). This HLM model with measurement points concerning mathematics enjoyment only was then run to generate a pattern of effects of the five measurement points for the identification of turning points. Finally, control variables, student characteristics at the student level and school characteristics at the school level, were introduced. Descriptive statistics of these student-level and school-level variables (used as control variables) are reported in Appendix B. If the previous HLM model functioned to demonstrate turning points in an absolute nature (i.e., without the presence of control variables), then the final HLM model functioned to demonstrate turning points in a relative nature (i.e., with the presence of control variables).

6. Results

Table 1 presents the results of the effects of each measurement point after adding the variable of mathematics enjoyment to the null HLM model without considering student and school characteristics. Obviously, each measurement point was treated to have random effects.

Table 1. Results of a Completely Nonlinear Approach of Multilevel Regression for Turning Points Concerning Effects of Mathematics Enjoyment on Mathematics Achievement, without Control of Student-Level Variables and School-Level Variables

	Fixed Effects		Random Effects	
	Effects	SE	Variance	χ^2
Mathematics enjoyment (ME) on mathematics achievement				
Measurement point: ME = 1	0.13*	0.03	0.19*	796.59
Measurement point: ME = 2	0.22*	0.04	0.22*	938.66
Measurement point: ME = 3	0.34*	0.04	0.23*	875.97
Measurement point: ME = 4	0.57*	0.03	0.21*	907.89
Measurement point: ME = 5	0.60*	0.03	0.16*	584.92

Note. * $p < 0.05$. For χ^2, $df = 231$.

All fixed effects were statistically significant and together showed a non-linear, positive, and increasing pattern of effects. Specifically, X had an effect of .13 on Y at X = 1; X had an effect of .22 on Y at X = 2; X had an effect of .34 on Y at X = 3; X had an effect of .57 on Y at X = 4; and X had an effect of .60 on Y at X = 5. Evidently, a turning point stood out at X = 4 (i.e., one positive change pattern occurred across X = 1, 2, and 3; and the other positive change pattern occurred across X = 4 and 5). We turned to the idea of an Electrocardiogram or EKG graph to illustrate this turning point (this EKG graph is for illustration only and may not be considered correct from the perspective of statistical graphing). An EKG graph is an effective way to identify different behavioral patterns. In Figure 1, it is evident that there was a jump in effects from X = 3 to X = 4 (i.e., the effects were relatively small across X = 1, 2, and 3 but the effects were relatively large across X = 4 and 5). Overall, a turning point was identified at X = 4, indicating that from X = 4 onward there were relatively larger effects.

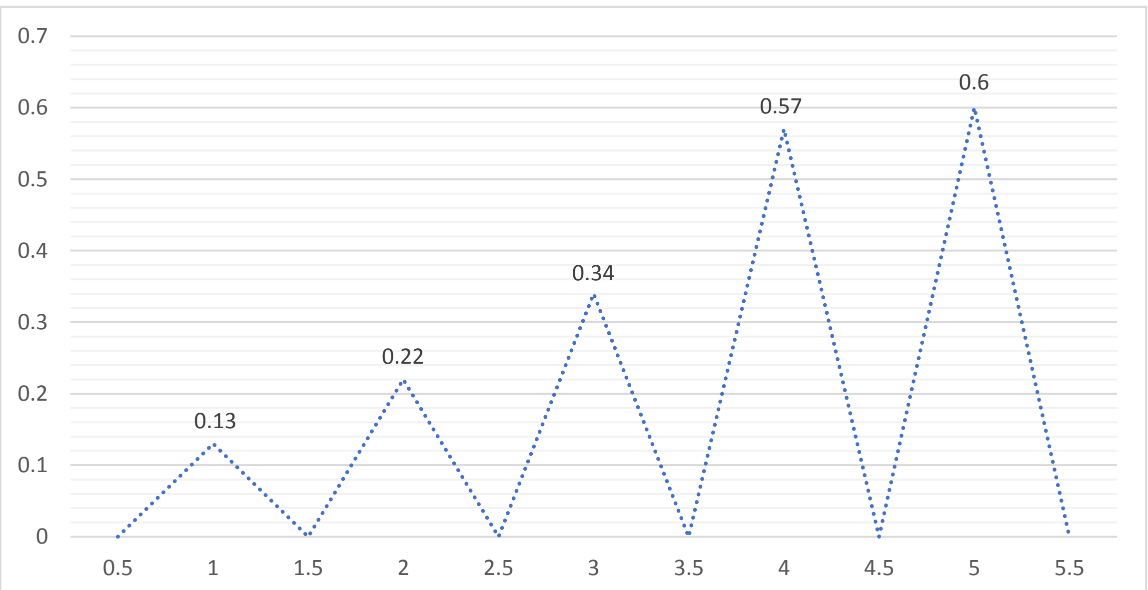

Figure 1. An EKG graph showing the trend of effects of mathematics enjoyment on mathematics achievement, without control of student characteristics and school characteristics

It was also informative to examine the random effects of the five measurement points (see Table 1). At each measurement point, the variance measured how varying the effects were across schools (i.e., the second-level units). It appeared that the effects at each measurement point varied statistically significantly across schools. It was worth noting that the identification of the turning point above was done permitting these random effects (variances). This cannot be easily done with the traditional approach of identifying turning points. Again, we believed that the random effects associated with the measurement points were informative to knowledge.

We note that the above model was referred to as the absolute model (without control of student-level and school-level variables) earlier. Student and school characteristics were then introduced to the absolute model as control variables at different levels, resulting in the relative model (with control of student-level and school-level variables) (see Table 2).

Table 2. Results of a Completely Nonlinear Approach of Multilevel Regression for Turning Points Concerning Effects of Mathematics Enjoyment on Mathematics Achievement, with Control of Student-Level Variables and School-Level Variables

	Fixed Effects		Random Effects	
	Effects	SE	Variance	χ^2
Mathematics enjoyment (ME) on mathematics achievement				
Measurement point: ME = 1	0.23*	0.11	0.18*	212.02
Measurement point: ME = 2	0.30*	0.09	0.17*	218.21
Measurement point: ME = 3	0.35*	0.10	0.23*	226.22
Measurement point: ME = 4	0.67*	0.09	0.19*	229.24
Measurement point: ME = 5	0.66*	0.09	0.19*	198.81

Note. * $p < 0.05$. For χ^2, $df = 152$.

Both fixed and random effects remained statistically significant even after controlling for student-level and school-level variables. Again, allowing effects at each measurement point to vary (i.e., adjusting for random effects or variance of effects at each measurement point), a turning point occurred at X = 4, or more precisely the turning point identified in the absolute model remained in the relative model. Similar to the case of the absolute model, an EKG graph was used to illustrate the different patterns of effects before and after the turning point at X = 4 (see Figure 2).

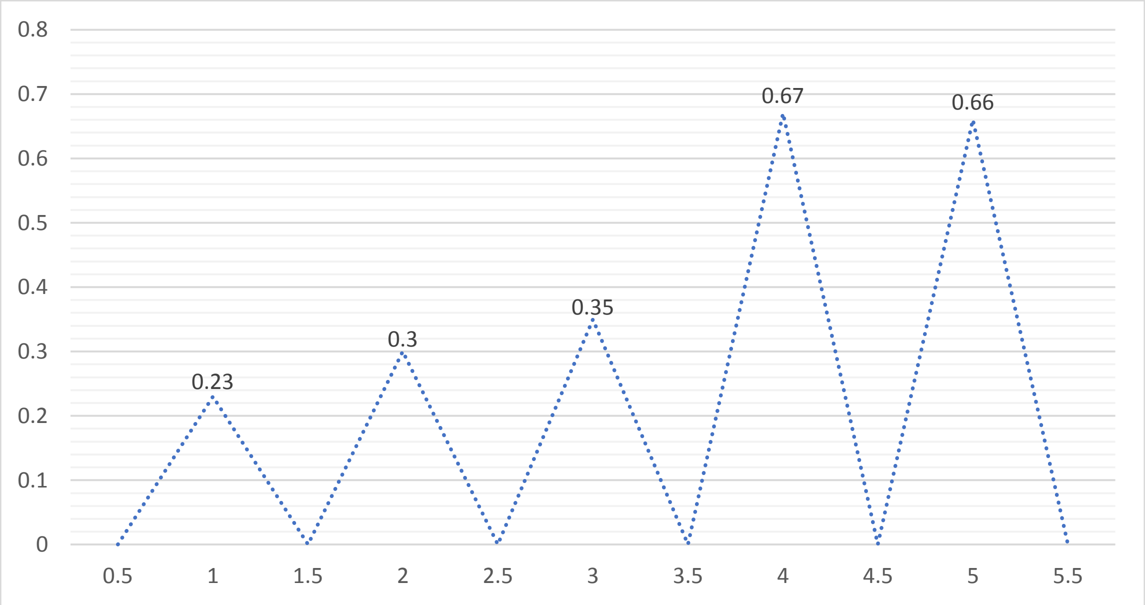

Figure 2. An EKG graph showing the trend of effects of mathematics enjoyment on mathematics achievement, with control of student characteristics and school characteristics

We also carried out what we referred to as comparison of neighborhood effects (on the HLM8.0 platform). This comparison aims to test whether the effects are statistically significantly different between two neighboring measurement points (e.g., between X = 1 and X = 2). This test is important as part of the evidence for the identification of a turning point. Table 3 presents the results of such an examination. For the absolute model (without control of student-level and school-level variables), the first three pairs of comparisons were statistically significant.

Table 3. Comparison of Effects of Mathematics Enjoyment on Mathematics Achievement between Neighboring Measurement Points for Identification of Turning Points, without and with Control of Student-Level Variables and School-Level Variables

	Without Control		With Control	
	t	SE	t	SE
X = 1 vs. X = 2	0.09*	0.02	0.07	0.09
X = 2 vs. X = 3	0.13*	0.03	0.05	0.09
X = 3 vs. X = 4	0.22*	0.03	0.32*	0.10
X = 4 vs. X = 5	0.03	0.03	0.00	0.09

* $p < 0.05$.

For example, the effects at X = 2 were statistically significantly different from the effects at X = 1. More importantly, the absolute model did indicate that the effects at X = 4 were statistically significantly different from the effects at X = 3, thus confirming the turning point at X = 4. Meanwhile, the relative model (with control of student-level and school-level variables) was even more revealing in that there was only one pair of comparisons statistically significant. This pair precisely identified or confirmed the turning point at X = 4 (i.e., the effects at X = 4 were statistically significantly different from the effects at X = 3).

It was quite evident that comparisons of neighborhood effects gave a clear confirmation of X = 4 as the turning point. Overall, mathematics enjoyment indicated positive effects on mathematics achievement at every measurement point. This is good confirmation that mathematics enjoyment did have positive effects on mathematics achievement. However, for the turning point to occur (i.e., for mathematics enjoyment to have strong effects on mathematics achievement), it appears that students must enjoy (doing) mathematics very much, implied by X = 4 on a measurement scale of 1 to 5.

Finally, we intended to come up with an assessment on the adequacy of our absolute and relative models. To do so, we calculated for each model the proportion of variance explained by the model at each measurement point. We note that this proportion does not concern the intercept (the proportion concerning which cannot be calculated because of the presence of the random slopes). Precisely speaking, this proportion concerns the (random) slopes (i.e., variance in slope associated with a certain measurement point). Table 4 presents the assessment on the adequacy of our absolute and relative models. For example, the relative model explained about 66% of the variance in slope (i.e., variance in effects) at the measurement point of X = 1. Overall, it was clear that the proportion of variance explained was quite adequate across all measurement points in each model (absolute or relative).

Table 4. Proportion of Variance Explained at Each Measurement Point

	Without Control	With Control
Measurement point: X = 1	0.64	0.66
Measurement point: X = 2	0.59	0.67
Measurement point: X = 3	0.56	0.57
Measurement point: X = 4	0.61	0.64
Measurement point: X = 5	0.70	0.64

7. Strengths and Limitations

7.1 Summary of Strengths

As an alternative analytical platform to the traditional piecewise regression for detecting turning points, this completely non-linear approach of multilevel regression demonstrates several advantages. We have alluded to its analytical advantages here and there when we introduced this approach earlier. Here is a summary and some extensions. First of all, this approach is multilevel, taking into account data hierarchy (e.g., students nested within schools) that can have critical influences on the identification of turning points at the student level. The multilevel nature of this approach allows the effects at each measurement point to vary (across schools), which brings researchers closer to reality (i.e., real-world condition). Under this approach, turning points can be identified with adjustment for variance of effects at each measurement point. Second, comparison of neighborhood effects provides confirmatory insights beyond any identification of qualitative nature (e.g., eyeball test). Confirmation is a necessary step in the identification of a turning point. Third, this approach can identify multiple turning points easily, which may be considered as a substantial advantage over the traditional approach that is not easy as a way to identify multiple turning points. In this study, we identified only one turning point because there was only one turning point. In the presence of two or more turning points, their identification can be organically or naturally done with this approach, and then can be followed with comparison of neighborhood effects for conformation. Such an advantage of this approach becomes obvious when there are a large number of measurement points.

Fourth, this approach can work with both continuous and categorical variables as candidates for turning points. This approach is a direct application in the case of a categorical variable. It is not easy for the traditional approach, on the other hand, to handle a categorical variable as a potential turning point. Although, in the case of a continuous variable, collapsing the continuous variable into categories has its problems, as we argued earlier, there are cases in reality where categorical variables are in fact preferred over continuous variables (e.g., for prevention of a turning point falling outside the range of legitimate values for a variable). In addition, this approach offers flexibility for researchers to define categories from a continuous variable according to their research intentions or needs (e.g., the purposeful choice of a certain region as the reference). Fifth, this approach allows covariates to be controlled separately at different levels (e.g., at student and school levels) so that the identification of turning points can be "refined" with adjustment for confounding effects at different levels. Finally, this approach is quite simple for researchers to apply. It is convenient for preparing data (i.e., reducing data management effort), and meanwhile, the HLM8.0 software provides an easy but complete way to operationalize this approach (i.e., model estimation and comparison of neighborhood effects).

7.2 Major Limitation and Future Research

In this approach, a continuous variable must be converted into a categorical variable for identification of turning points. As we alluded to earlier, this fact is both positive and negative. The positive side is that researchers can simplify a continuous variable by highlighting the most important regions for the examination of effects based on their research purposes. Oftentimes, "categorical effects" are not only easier to interpret by researchers but also easier to understand by educators, parents, administrators, and policymakers, compared with "continuous effects." The negative side is that there is the loss of information when converting a continuous variable into a categorical variable. The fact that a candidate variable for turning points must be a categorical variable in order to apply this approach is perhaps the major (inherited) limitation.

From the model development perspective, this approach is univariate, meaning that it tested only one outcome measure or one dependent variable. The multilevel piecewise regression in a multivariate fashion is not yet available in the literature. This will be a good contribution of the current approach in the future. Multilevel, multivariate piecewise regression will be a natural extension of the current approach, having the ability to test, for example, two school subjects (e.g., mathematics and science) simultaneously for similarities and differences in terms of turning points of the effects of enjoyment on achievement between the two school subjects.

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Appendix A. Description of Composite Variables

Mathematics Enjoyment (A Student-Level Variable)
How much do you agree with the statements about learning mathematics? (a) I enjoy learning mathematics. (b) I wish I did not have to study mathematics. (c) Mathematics is boring. (d) I learn many interesting things in mathematics. (e) I like mathematics. (f) I like any schoolwork that involves numbers. (g) I like to solve mathematics problems. (h) I look forward to mathematics lessons. (i) Mathematics is one of my favorite subjects. (Response: very much, somewhat, not much) (Reliability = .93)
School Resources (A School-Level Variable)
To what extent is mathematics instruction affected by shortage of following resources? (a) Teachers with a specialization in mathematics. (b) Computer software/applications for mathematics instruction. (c) Library resources relevant to mathematics instruction. (d) Calculators for mathematics instruction. (e) Concrete objects or materials to help students understand quantities or procedures. (Response: not affected, somewhat affected, affected a lot) (Reliability = .93)
Academic Pressure (A School-Level Variable)
How would you characterize each of the following within your school? (a) Teachers' understanding of the school's curricular goals. (b) Teachers' degree of success in implementing the school's curriculum. (c) Teachers' expectations for student achievement. (d) Teachers' ability to inspire students. (e) Parental involvement in school activities. (f) Parental commitment to ensure that students are ready to learn. (g) Parental expectations for student achievement. (h) Parental support for student achievement. (i) Students' desire to do well in school. (j) Students' ability to reach school's academic goals. (k) Students' respect for classmates who excel academically. (Response: very high emphasis, high emphasis, medium emphasis) (Reliability = .92)
School Disciplinary Climate (A School-Level Variable)
To what degree is each of the following a problem among (fourth grade) students in your school? (a) Arriving late to school. (b) Absenteeism (i.e., unjustified absences). (c) Classroom disturbance. (d) Cheating. (e) Profanity. (f) Vandalism. (g) Theft. (h) Intimidation or verbal abuse among students (including texting, emailing, etc.). (i) Physical fights among students. (j) Intimidation or verbal abuse of teachers or staff (including texting, emailing, etc.) (Response: hardly any problems, minor problems, moderate to severe problems) (Reliability = .89)

Appendix B. Descriptive Statistics of Student-Level and School-Level Variables

Variables	Mean	SD
Student-level variables		
Mathematics enjoyment (divided into measurement points)	9.68	2.18
0 to 20th percentile (3.85 to 7.98)	6.75	1.23
20 to 40th percentile (7.98 to 8.96)	8.57	0.28
40 to 60th percentile (8.96 to 9.97)	9.53	0.30
60 to 80th percentile (9.98 to 11.74)	10.97	0.56
80 to 100th percentile (11.76 to 13.14)	13.13	0.08
Gender (male = 0, female = 1)	0.49	0.01
Age (continuous)	10.25	0.43
Socioeconomic status (SES)		
Less books at home (vs. median number of books at home)	0.42	0.49
More books at home (vs. median number of books at home)	0.26	0.44
Home language (English = 1, others = 0)	0.64	0.48
Immigration status (yes = 1, no = 0)	0.93	0.27
School-level variables		
School location		
Urban (vs. rural)	0.19	0.39
Suburban (vs. rural)	0.51	0.50
School socioeconomic composition		
(Disadvantaged proportion: > 50% = 1, ≤ 50% = 0)	0.61	0.49
School racial-ethnic composition		
(Majority proportion: > 50% = 1, ≤ 50% = 0)	0.79	0.41
School resources (for mathematics education)	11.38	2.35
Academic pressure	10.05	2.38
School disciplinary climate	9.85	1.47

Note. For dummy variables, a mean indicates the proportion of cases coded as 1.